

Different faces of the phantom

K.A. Bronnikov[‡], J.C. Fabris[§] and S.V.B. Gonçalves^{||}

Departamento de Física, Universidade Federal do Espírito Santo, Vitória, ES, Brazil

Abstract. The SNe type Ia data admit that the Universe today may be dominated by some exotic matter with negative pressure violating all energy conditions. Such exotic matter is called *phantom matter* due to the anomalies connected with violation of the energy conditions. If a phantom matter dominates the matter content of the universe, it can develop a singularity in a finite future proper time. Here we show that, under certain conditions, the evolution of perturbations of this matter may lead to avoidance of this future singularity (the Big Rip). At the same time, we show that local concentrations of a phantom field may form, among other regular configurations, black holes with asymptotically flat static regions, separated by an event horizon from an expanding, singularity-free, asymptotically de Sitter universe.

1. Introduction

The evidence for an accelerating expanding phase of the universe seems to be robust [1]. If this is the case, the deceleration parameter $q = -\ddot{a}a/\dot{a}^2$, must be negative, implying that the matter dominating the matter content of the universe, if described in terms of a perfect fluid, must have an equation of state $p = w\rho$, with $w = -1/3$. Hence, the strong energy condition must be violated. More recently, there has been claims that the observational data favour an equation of state with $w < -1$ [2]. Matter with such an equation of state violates all the energy conditions. This kind of matter can be represented, in a more fundamental way, by a self-interacting scalar field, whose kinetic energy appears with the "wrong sign". For this reason, it is more popularly called a *phantom field*. In the usual hydrodynamics representation, a matter with this equation of state is unstable. However, such an instability may disappear when a fundamental description is employed, for example, using a self-interacting scalar field, as stated before.

If phantom matter dominates the matter content of the Universe, a future singularity may develop in a finite proper time since its density grows as the universe expands. Such a singularity inevitably appears in an isotropic Universe if $w = \text{const} < -1$. This possible future singularity has been named a *Big Rip*, leading to the notion of

[‡] E-mail: kb20@yandex.ru. Permanent address: Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow, Russia; Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia.

[§] E-mail: fabris@cce.ufes.br

^{||} E-mail: sergio@cce.ufes.br

a *phantom menace* [3]. However, it must be stressed that the possibility that phantom matter dominates the universe today is yet a matter of debate: the observational data, mainly those coming from the supernovae type Ia of high redshift, lead to different conclusions depending on how the sample is selected, and even how the statistical analysis is performed [4]. In any case, the possibility that a phantom matter has something to do with the actual universe must be taken seriously.

The goal of the present work is twofold. First, we intend to analyse the evolution of scalar perturbations for phantom matter. This is an important point, since we are dealing with matter with negative pressure, and instabilities may develop, mainly at small scales [5]. It is possible to get rid of these instabilities if a scalar field representation is used: the behaviour of the perturbations on small scales are quite sensitive to the description used for the matter [6]. On the other hand, the behaviour of the perturbations at large scales are quite insensitive to the description employed. We will show, using a self-interacting scalar field simulating a perfect fluid with $w = \text{const}$, that phantom matter may lead to growing perturbations at large scales if the pressure is negative enough. This may lead to a very inhomogeneous universe deep in the phantom era, and such inhomogeneities may lead to avoidance of a Big Rip.

Besides this perturbative analysis of a cosmological scenario where the phantom matter dominates the matter content of the universe, we will study local configurations with spherical symmetry. In this case, the results are still more unexpected. The fact that the kinetic term has a "wrong" sign, may lead to a minimum of the radius of coordinate 2-spheres, so that a central singularity is avoided by having no center at all, as is the case with wormholes. However, such configurations may contain one or two Killing horizons, and, among others, it is possible to have configurations where there is a static, asymptotically flat region which is separated by an event horizon from an expanding singularity-free, Kantowski-Sachs type universe. It is thus a black hole in which an explorer may survive after crossing the horizon.

2. Evolution of scalar perturbations

When a barotropic fluid with the equation of state $p = w\rho$ is introduced in the Einstein's equations, with a Friedmann-Robertson-Walker flat line element $ds^2 = a^2(d\eta^2 - dx^2 - dy^2 - dz^2)$, η being the conformal time, the conservation equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad , \quad (1)$$

leads, in case $w = \text{const}$, to $\rho \propto a^{-3(1+w)}$. Inserting this into the Friedmann equation

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho a^2 \quad , \quad (2)$$

we find that the scale factor behaves as $a \propto \eta^{\frac{2}{1+3w}}$. One important feature of this solution that must be stressed in order to understand the behaviour at perturbative level, is the character of the "future". If $w > -1/3$, when all energy conditions hold, $\eta \rightarrow \infty$ means $a \rightarrow \infty$; when $w < -1/3$, on the other hand, the universe is expanding as $\eta \rightarrow 0_-$.

In general, fluids with negative pressure contain, at a perturbative level, decreasing modes at large scales and unstable models at small scales [5]. The instabilities at small scales must not be taken so seriously, since it must be due mainly to the hydrodynamical approximation [6]. It is possible to use a more fundamental representation for such exotic matter by considering a self-interacting scalar field, which reproduces, from the point of view of the background behaviour, the hydrodynamical approach employed until now. A scale factor which evolves as $a \propto \eta^{\frac{2}{1+3w}}$, with $w < -1$, can be achieved by considering a self-interacting minimally coupled scalar field, such that,

$$V(\phi) = V_0 \exp\left(\pm \sqrt{-3(1+w)}\phi\right) \quad , \quad \phi = \pm 2 \frac{\sqrt{-3(1+w)}}{1+3w} \ln \eta \quad . \quad (3)$$

A similar model can be constructed when $w > -1$ by just changing the sign of the term inside the square roots.

For gravity minimally coupled to a (self-interacting) scalar field, the equations for the perturbed quantities reduce to a single equation for the metric perturbed function Φ , called Bardeen's potential, which is [7]

$$\Phi'' + 2\left\{H - \frac{\phi''}{\phi'}\right\}\Phi' + \left\{k^2 + 2\left[H' - H\frac{\phi''}{\phi'}\right]\right\}\Phi = 0 \quad . \quad (4)$$

Using the background expressions for H and ϕ , this equation becomes,

$$\Phi'' + 2\frac{3(1+w)}{1+3w}\frac{\Phi'}{\eta} + k^2\Phi = 0 \quad , \quad (5)$$

with the solutions

$$\Phi = (k\eta)^{-\nu} \left\{ c_1(k)J_\nu(k\eta) + c_2(k)J_{-\nu}(k\eta) \right\} \quad , \quad \forall w \quad , \quad (6)$$

where $\nu = (5+3w)/[2(1+3w)]$ and k is the wavenumber of the perturbations, resulting from a plane wave expansion of the spatial part of the perturbed quantities.

In the small-scale asymptotic limit defined by $k\eta \ll 1$, the solutions behave as in the hydrodynamical representation:

$$\Phi \propto c_1 + c_2(k\eta)^{-2\nu} \quad . \quad (7)$$

In all cases, there is a constant mode. However, if $w > -5/3$ the second mode decreases as the universe expand; but there is a growing mode when $w \leq -5/3$, what can lead to formation of large inhomogeneties. There is an asymptotic logarithmic divergence for $w = -5/3$. Using, on the other hand, the asymptotic expression for the Bessel functions for large values of the argument $k\eta \gg 1$, the potential can be expressed as

$$\Phi \sim (k\eta)^{-\frac{1+w}{1+3w}} \cos(k\eta + \delta) \quad , \quad (8)$$

δ being a phase. It is easy to verify that for $w > -1$, the potential oscillates with decreasing amplitude, while for $w < -1$, the potential oscillates with increasing amplitude. Hence, for $w < -5/3$ the phantom field may exhibit instability at large and small scales. However, it must be stressed that the behaviour at small scale is quite model-dependent, and another field representation of the phantom field can modify the conclusions at small scales, like considering the phantom field as a ghost condensation

[8] or a tachyon [9]. But, at large scales, it seems that there is always a growing mode, for $w \leq -5/3$, irrespective of the representation chosen.

It is fundamental now to understand the meaning of small and large asymptotic limits, $k\eta \ll 1$ or $k\eta \gg 1$ respectively. We normalize the scale factor by fixing $a_0 = 1$ at the present time. This implies that the Hubble parameter, expressed in terms of the cosmic time, is given by $\mathcal{H}_0 = \frac{\dot{a}}{a}|_{t=t_0} = \frac{a'}{a^2}|_{\eta=\eta_0} = \frac{2}{|1+3w|} \frac{1}{|\eta_0|}$, where η_0 is the conformal time today. Hence, $\eta_0 \sim \mathcal{H}_0^{-1} = l_H$ (with $c = 1$). This implies that the separation point between the large and small scale regimes is given by the Hubble length l_H . When $w > -1/3$, the conformal time increases as the Universe expands, implying that, as time goes on, more and more modes satisfy the condition $k\eta \gg 1$, which can be re-expressed by saying that the modes enter the Hubble horizon as time goes on. The opposite occurs when $w < -1/3$, and as time goes on, more and more physical modes satisfy the condition $k\eta \ll 1$, the usual situation of an accelerated expansion phase. In the interval $-5/3 < w < -1/3$, these modes that are stretched outside the Hubble horizon are frozen or decay, and there is no danger for homogeneity. But, for $w < -5/3$, these modes begin to become strongly unstable and homogeneity can be destroyed. The above results can be re-expressed using, as a reference parameter, the Hubble horizon as a function of time. In fact, using the expression for the Hubble length at any time, we have $l_H(\eta) = \frac{|1+3w|}{2} |\eta|^{\frac{3(1+w)}{1+3w}} |\eta_0|^{\frac{-2}{1+3w}}$ and the argument of the Bessel functions written above can be expressed as $k\eta \sim k [l_H(\eta)]^{\frac{1+3w}{3(1+w)}}$. For the phantom field, $l_H(\eta)$ decreases as the Universe expands. Hence, for $w < -5/3$ more and more modes go out of the Hubble horizon and begins to grow, enhancing the inhomogeneity. Notice that for $w = -1$ (cosmological constant case), the Hubble horizon remains constant.

Even if the calculations exposed here were done for the flat case, they also apply to the open and closed cases [10]. These results have been obtained with the potential (3) that reproduces the equation of state $w = \text{const}$ in an isotropic universe. It should be mentioned that other forms of the potential may not lead to a Big Rip even in the isotropic case: e.g., if V is bounded above, a phantom-dominated universe evolves, in general, toward a de Sitter attractor solution [12].

3. Local configurations

Let us consider now the Hilbert-Einstein Lagrangian coupled to a self-interacting scalar field with an unspecified sign for the kinetic term. For the moment, we do not specify the potential. Considering the spherically symmetric metric written in the form

$$ds^2 = A dt^2 - A^{-1} d\rho^2 - r(\rho)^2 d\Omega^2 \quad , \quad (9)$$

ρ being the radial variable, we find the following set of coupled differential equations:

$$(A r^2 \phi')' = \epsilon r^2 V_\phi \quad , \quad (10)$$

$$(A' r^2)' = -2V r^2 \quad , \quad (11)$$

$$2r''/r = -\epsilon \phi'^2 \quad , \quad (12)$$

$$(r^2)'' A - A'' r^2 = 2 \quad . \quad (13)$$

If $\epsilon = 1$ we have a "normal" scalar field, while if $\epsilon = -1$ we have a phantom scalar field. Equation (13) is once integrated giving

$$(A/r^2)' = 2(\rho_0 - \rho)/r^4, \quad \rho_0 = \text{const} \quad . \quad (14)$$

We will summarize the possible configurations in what follows without solving explicitly the equations (10-13) [11].

Let us indicate the possible kinds of nonsingular solutions without restricting the shape of $V(\phi)$. Assuming no pathology at intermediate ρ , regularity is determined by the system behavior at the ends of the ρ range. The latter may be classified as a regular infinity ($r \rightarrow \infty$), which may be flat, de Sitter or AdS, a regular center $r \rightarrow 0$, and the intermediate case $r \rightarrow r_0 > 0$. Suppose we have a regular infinity as $\rho \rightarrow \infty$, so that $V \rightarrow V_+ = \text{const}$ while the metric becomes Minkowski (M), de Sitter (dS) or AdS according to the sign of V_+ . In all cases $r \approx \rho$ at large ρ .

For $\epsilon = +1$, due to $r'' \leq 0$, r necessarily vanishes at some $\rho = \rho_c$, which means a center, and the only possible regular solutions interpolate between a regular center and an AdS, flat or dS asymptotic; in the latter case the causal structure coincides with that of de Sitter space-time.

For $\epsilon = -1$, there are similar solutions with a regular center, but due to $r'' \geq 0$ one may obtain either $r \rightarrow r_0 = \text{const} > 0$ or $r \rightarrow \infty$ as $\rho \rightarrow -\infty$. In other words, all kinds of regular behavior are possible at the other end. In particular, if $r \rightarrow r_0$, we get $A \approx -\rho^2/r_0^2$, i.e., a cosmological region comprising a highly anisotropic Kantowski-Sachs cosmology (KS) with one scale factor (r) tending to a constant while the other (A) inflates. The scalar field tends to a constant, while $V(\phi) \rightarrow 1/r_0^2$.

Thus there are three kinds of regular asymptotics at one end, $\rho \rightarrow \infty$ (M, dS, AdS), and four at the other, $\rho \rightarrow -\infty$: the same three plus $r \rightarrow r_0$, simply r_0 for short. (The asymmetry has appeared since we did not allow $r \rightarrow \text{const}$ as $\rho \rightarrow \infty$. The inequality $r'' > 0$ forbids nontrivial solutions with two such r_0 -asymptotics.) This makes nine combinations shown in Table 1. Moreover, each of the two cases labelled KS* actually comprises three types of solutions according to the properties of $A(\rho)$: there can be two simple horizons, one double horizon or no horizons between two dS asymptotics. Recalling 3 kinds of solutions with a regular center, we obtain as many as 16 qualitatively different classes of globally regular configurations of phantom scalar fields.

Table 1. Regular solutions with for $\epsilon = -1$. Each row corresponds to a certain asymptotic behavior as $\rho \rightarrow +\infty$, each column — to $\rho \rightarrow -\infty$. The mark "sym" refers to combinations obtained from others by symmetry $\rho \leftrightarrow -\rho$.

	AdS	M	dS	r_0
AdS	wormhole	wormhole	black hole	black hole
M	sym	wormhole	black hole	black hole
dS	sym	sym	KS*	KS*

4. Conclusions

Phantom matter implies violation of all energy conditions. In a hydrodynamical representation, phantom matter is described by $p = w\rho$ with $w < -1$. Phantom matter can be described by a self-interacting scalar field with a "wrong" sign in the kinetic term. There is some evidence that the exotic matter responsible for the actual phase of accelerated expansion of the universe may be a kind of phantom field. In this work, we have studied some properties of a phantom field, specifically with respect to the evolution of scalar perturbations and with respect to local configurations.

We found that, under certain conditions, a universe dominated by a phantom matter may develop high inhomogeneities even at large scales. Hence, after a certain stage of its evolution, the hypothesis of homogeneity and isotropy becomes no more valid. As the big rip scenario depends on these hypothesis, it is possible that a phantom universe brings in itself a mechanism of avoiding a future singularity even in the case when $w = \text{const}$ in homogeneous and isotropic space-time.

In what concerns local configurations, we find a wealth of nonsingular models among which of particular interest are asymptotically flat black holes with an expanding universe beyond the event horizon. This provides an interesting singularity-free cosmological scenario: one may speculate that our Universe could appear from collapse to such a phantom black hole in another, "mother" universe and undergo isotropization (e.g., due to particle creation) soon after crossing the horizon.

There are no similar configurations with a "normal" scalar field. In any case, violation of all energy condition inevitably leads to completely new configurations.

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